

Modeling Work-Flow of the “Cylinder-Piston” Type Devices Using a Universal Thermodynamic Model

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Abstract

A universal thermodynamic model for calculating the workflow of “cylinder-piston” type systems which describes step by step the processes in the cylinder, and takes into account the real nature of the intake-exhaust and air leakage processes has been developed. Using the model, three completely different applied problems, including modeling hydrolock when fluid enters the cylinder of an internal combustion engine, modeling the dynamics of the catapult trolley motion during the launch of an unmanned aerial vehicle and modeling the engine indicator diagram during cold cranking when measuring compression were solved. Control modeling and comparison with known experimental data on the processes of air compression in cylinder with liquid (hydrolock), results of pressure measures in the engine cylinder during cold cranking and the known data of pneumatic launchers (catapults) showed the model’s overall reliability and its applicability for various applied tasks. This determines the model for possible use not only in designing devices of the type under consideration, but also for their operational diagnosing and troubleshooting of various types of damage.

Keywords: Thermodynamic model; Modeling; Cold cranking; Compression; Hydrolock; Diagnostics; Pneumatic launcher

Abbreviation: M, m: Mass; V, L: Volume and Length; F, f: Area; r: Relative Length, Radius; H, h: Height; S, x: Stroke (Piston) and Current Coordinate; τ : Current Time; φ , n: Rotation Angle and Rotation Speed; p, T: Pressure and Temperature; C, v: Velocity; T, R, F: Force; U, i: Internal Energy and Enthalpy; Q, L: Amount of Supplied (Removed) Heat and Work; C_v, C_p: Heat Capacities at Constant Pressure and Volume; γ , R: Air Adiabatic Index and Gas Constant; ϵ : Compression Ratio, Relative Volume; μ : Flow Coefficient

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Introduction

In technology, numerous devices are used, the principle of operation of which is based on the work of a piston in a cylinder. These are not only piston internal combustion engines and pneumatic compressors, but also various pneumatic actuators and drives. In general, all these devices can be considered by completely different groups of scientific schools and researchers, but at the same time have a very close theoretical description. This makes it possible to create a certain general and universal mathematical model that could be applied to such completely different types of structures and devices. The advantage of this approach is that it allows you not only to generalize the mathematical description of similar processes, but also to perform mathematical modeling where it is not used for one reason or another.

Analysis of the latest research and publications

Thermodynamic modeling of processes [1] is currently used mainly at the stages of preliminary design, selection of basic parameters and study of the characteristics of various piston-type devices. Such modeling is also called zero-dimensional [2], since it is assumed that the instantaneous parameters of the working fluid (gas or air) have a uniform distribution

over the device control volume, and the velocity of the working fluid inside this volume is not taken into account. Among the devices of this type, the largest group is made up of internal combustion engines [3], for the modeling of which special computer programs have been developed. For example, the well-known LES software (Lotus Engine Simulation [4]) is completely based on the thermodynamic model of the cylinder. The GT-Power software also considers the thermodynamic process in the cylinder and, in some sources [5], is considered one of the most advanced in describing intra-cylinder processes. More complex software uses a similar thermodynamic description of the process in the cylinder, although they provide for the possibility of improving individual elements (models) included in the software, including by increasing their dimensionality. The widely known AVL-Boost software [6] is based on this principle. Another well-known software, Ricardo-WAVE [7], considers a similar thermodynamic representation of the process in the cylinder, but also allows the addition of 3-dimensional user models to clarify the mixing and combustion processes in the cylinder. In other words, thermodynamic models are not only traditionally the basis of well-known software for modeling internal combustion engines, but are also well-developed for practical application [8].

However, the situation changes dramatically as soon as any non-standard task arises. For example, when studying the processes of starting internal combustion engines, not all known software is applicable to describing intra-cylinder processes at low piston velocity [9]. Under such conditions, heat exchange and mixture formation processes can differ significantly from what occurs in the engine in operating modes. As a result, we see a picture where researchers are forced to develop their own models each time to describe the same thermodynamic processes [10]. In [11], it is shown that the most typical situation has developed in studies of engine cold cranking during diagnostic compression measurements. In fact, such processes are currently not modeled, but are studied mainly using experimental [12,13] and probabilistic-statistical methods [14], which usually consider arrays of experimental data [15]. Such methods are effective for diagnosing the technical condition in operation [16], but are quite far from modeling in-cylinder processes of engines. So, cold cranking models used to study engine starting characteristics [17] are not applicable to the specific diagnostic problem of measuring compression, in which there is no fuel supply, no mixture formation and no combustion. This means that in the absence of experimental data and modeling, it is very difficult or even impossible to determine any patterns in the process or object under study.

On the other hand, the use of known models also becomes impossible when studying various engine failures. For example, such faults are known that are associated with the ingress of various liquids into the cylinder [18]. To describe the abnormal process of air compression with liquid (so-called hydrolock), it is necessary to take into account the volume of liquid and its effect on the parameters of the working fluid in the cylinder, which actually defines this mode as non-working. As a result, the models describing the working process of the engine cannot be used to simulate non-working

(non-design) modes and damage, which often occur when operating conditions are broken [19]. Similar difficulties are also observed when studying other piston-type devices. For example, the widely known type of pneumatic catapult for launching small-sized aircraft, as it turned out [20], is also practically not modeled. Most of the known works on the selection of parameters and study of such devices are based on the simplest analysis of the acting forces [21], which gives a very limited understanding of the processes taking place. At the same time, thermodynamic analysis of the control volume of any such device is not a complex task and does not require any special computing tools. However, when modeling them, results can be obtained that are difficult or even almost impossible to obtain only through experimental research and statistical models. This indicates that the use of thermodynamic modeling in the research, development, creation and operation of piston-type devices has practically no alternative.

Statement of the problem

In accordance with the above, the aim of the work is to develop a universal thermodynamic model of a cylinder-piston system applicable to various types of pneumatic devices.

To achieve this goal, it seems necessary to solve the following problems:

- to construct a general thermodynamic model of a piston device cylinder;
- to consider particular cases of piston devices of various types that require clarification of the mathematical description and determination of initial and boundary conditions within the framework of the model under consideration;
- to simulate all the devices under consideration and evaluate the reliability of the model created.

Methodology

Construction of a general thermodynamic model of piston-type devices

To create a thermodynamic model of a piston device, let us consider a cylinder in which a piston moves (Figure 1). To derive the model equations, we use the equation of state of an ideal gas in a cylinder:

$$p = \frac{M}{V} RT \quad (1)$$

and the equation of the 1st law of thermodynamics [1], written for the volume of a cylinder V:

$$dU = dQ - dL + idM \quad (2)$$

In equations (1) and (2): p , T are the pressure and temperature of air in the cylinder; $dU = d(MC_p T)$ the change in the internal energy of air in the cylinder; dQ is the amount of supplied (+) or removed (-) heat; $dL = pdV$ is the work of air; $i = C_p T$ the enthalpy of air; dM is the change in the mass M of air in the cylinder due to the inflow into the cylinder (+) or leakage from the cylinder (-); C_p ,

C_p are the heat capacities at constant pressure and volume, related by the formula $C_p = \gamma C_v = \gamma R / (\gamma - 1)$ where γ , R are the adiabatic index of air and the gas constant.

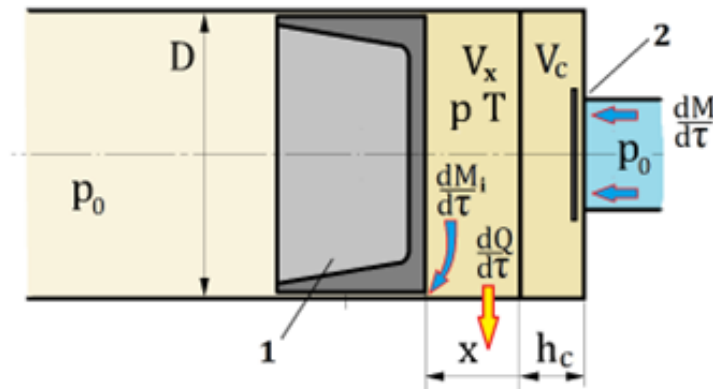


Figure 1: General diagram of the studied “cylinder-piston” system: 1-piston, 2-inlet/outlet valve.

The air volume in the cylinder is obviously determined by the area F_p and the piston position, equal to the distance x from the top dead center:

$$V = V_x + V_c = V_x + \frac{V_H}{\epsilon - 1} = V_H \left(\frac{x}{S} + \frac{1}{\epsilon - 1} \right) = F_p (x + H_c) = F_p l_x \quad (3)$$

where S is the piston stroke; V_H , V_c are the working and “dead” volume of the cylinder; $V_x = F_p x$ is the current volume opened by the piston; ϵ is the geometric compression ratio; $H_c = V_c / F_p = S / (\epsilon - 1)$ is the length of the “dead” volume (or combustion chamber); $l_x = x + H_c$ is the current working length of the cylinder. Considering that the change in the cylinder volume is $dV = F_p dx$ and the air temperature in the absence of combustion changes insignificantly, from equation (2) by substituting equation (1) into it we obtain a differential equation for the air temperature in the cylinder:

$$\frac{dT}{T} = \frac{(\gamma - 1)}{l_x} \left[-dx + \frac{RT}{F_p p} \left(dM + \frac{dQ}{RT} \right) \right] \quad (4)$$

On the other hand, after differentiating the equation of state (1), we obtain a differential equation for the air pressure in the cylinder:

$$\frac{dp}{p} = \frac{\gamma}{l_x} \left[-dx + \frac{RT}{F_p p} \left(dM + \frac{\gamma - 1}{\gamma} \frac{dQ}{RT} \right) \right] \quad (5)$$

If equations (4) and (5) are divided by the time increment $d\tau$ and heat exchange with the cylinder walls is neglected (for example, at low process temperatures, no combustion, rapid process, etc.), a mathematical model of the process under study can be obtained - a system of first-order differential equations resolved with respect to the derivative, in the form [11,19,20]:

$$\left\{ \begin{array}{l} \frac{dT}{d\tau} = (\gamma - 1) \psi T \\ \frac{dp}{d\tau} = \gamma \psi p \\ \psi = \frac{1}{l_x} \left(-\frac{dx}{d\tau} + \frac{RT}{F_p p} \frac{dM}{d\tau} \right) \end{array} \right. \quad (6)$$

However, before applying the resulting system of equations to solve the practical problems under consideration, it is necessary to set the boundary, initial and other conditions characteristic of each of these problems.

Particular cases of piston models

Let us consider some particular cases of piston operation in a cylinder, which allow us to formulate the desired conditions. The presence of a liquid in the cylinder, occupying a relative volume $\epsilon_v = v / V_c$ of the “dead” space of the cylinder, gives the following view of the expression (3):

$$V = V_H \left(\frac{x}{S} + \frac{1 - \epsilon_v}{\epsilon - 1} \right) = F_p [x + H_c (1 - \epsilon_v)] \quad (7)$$

If the working length of the cylinder, which is included in the system of equations (6), to write as $l_x = x + H_c (1 - \epsilon_v)$. The presence of a receiver, a pre-chamber or any other additional volume V_R attached to the cylinder allows expression (3) to be written as:

$$V = V_R + V_H \left(\frac{x}{S} + \frac{1 - \epsilon_v}{\epsilon - 1} \right) = V_R + F_p [x + H_c (1 - \epsilon_v)] \quad (8)$$

From which the cylinder working length, included in the system of equations (6), will be in the general case:

$$l_x = V_R / F_p + x + H_c (1 - \epsilon_v) \quad (9)$$

For practical application of the model, it is necessary to detail it by determining the patterns of change of the piston coordinate x and the air flow rate $dM/d\tau$, flowing into or out of the cylinder.

For a crank mechanism, in accordance with Figure 2a, the x coordinate can be expressed by the formula:

$$x = \frac{S}{2} \left(1 + r - \cos \phi - \sqrt{r^2 - \sin^2 \phi} \right) \quad (10)$$

where $r = 2L_c / S$ is relative length of connecting rod.

Differentiation of the equation (10) gives the piston accelera-

tion, which is included in the system of equations (6):

$$\frac{dx}{d\tau} = \frac{\pi S n}{60} \sin \phi \left(1 + \frac{\cos \phi}{\sqrt{r^2 - \sin^2 \phi}} \right) = v_m \frac{\pi}{2} \sin \phi \left(1 + \frac{\cos \phi}{\sqrt{r^2 - \sin^2 \phi}} \right) \quad (11)$$

Where $C_m = Sn/30$ is average piston velocity.

For the pulley mechanism of a catapult system, piston motion (Figure 2a & 2b) is determined by trolley position and cable length [21]:

$$x_p = L_c - \frac{H - r \sin \beta}{\sin \alpha} - \frac{\pi r (90^\circ + \beta)}{180} - l_0 - r \quad (12)$$

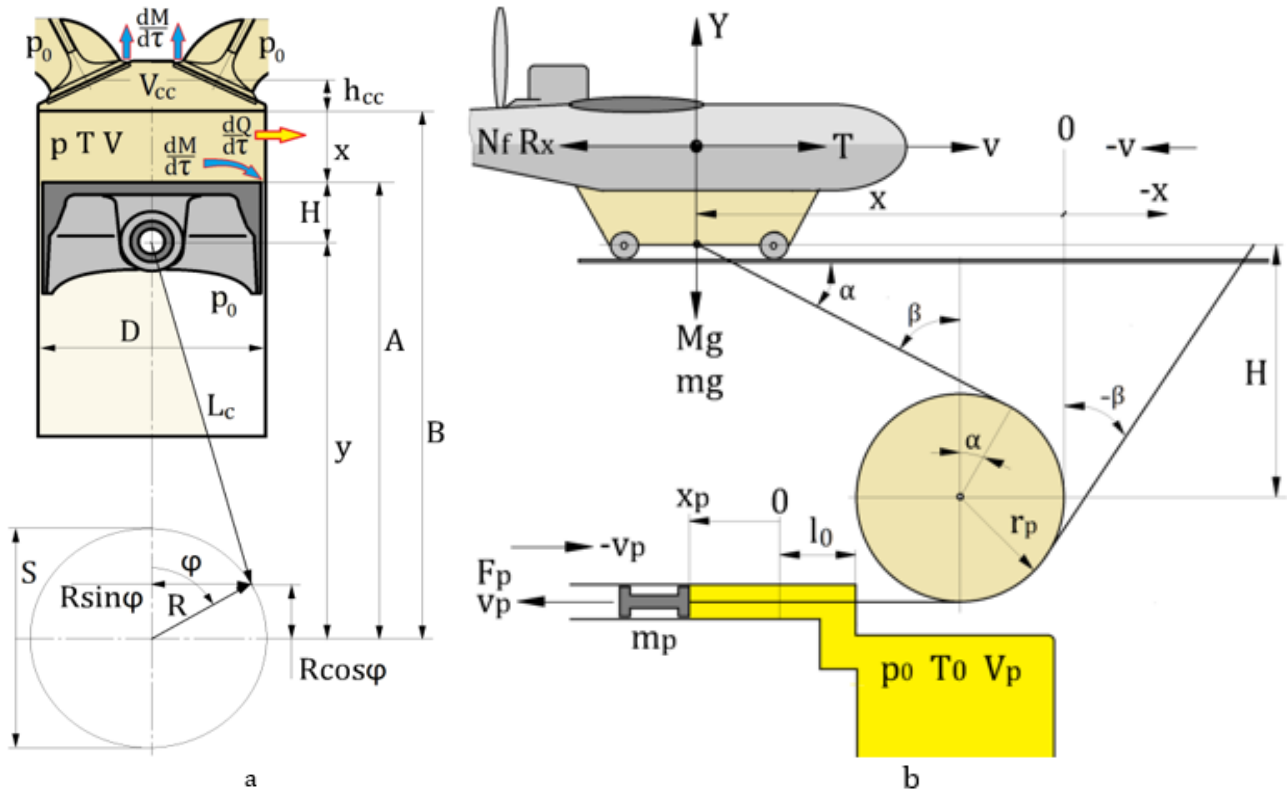


Figure 2: Examples of piston movement patterns: a-crank mechanism of an engine, b-pulley mechanism of a pneumatic catapult.

The velocities of the trolley v and the piston v_p are related by the ratio $v_p = v \sin \beta$ and the angle between the motion direction of the piston and the trolley can be found by the formula:

$$\beta = \arctg \frac{1}{H} \left[x + r \left(\sqrt{1 + \left(\frac{x}{H} \right)^2} - 1 \right) \right]$$

In this case, to determine the velocity, it is necessary to add another equation for the acceleration of the trolley to the system of equations (6):

$$\frac{dv}{d\tau} = \frac{F_{px} + T - R_x - F_f - F_{gx}}{M_\Sigma} \quad (13)$$

Where $M_\Sigma = M + m$ is the total mass of the accelerated system; M, m are the masses of the aircraft, the trolley and the cable connected to it; F_{px} is the projection on the x -axis of the cable traction force determined by the pressure p in the cylinder; T, R_x are the engine thrust and the aerodynamic drag force of the aircraft and trolley, F_f, F_{gx} are the friction force and the gravity projection on the x -axis (if the catapult has an angle of ascent). The change in the mass of air in

the cylinder is determined by air flow through the valves and leak points. When the pressure in the cylinder is higher than atmospheric, air flow from the cylinder through a certain opening (clearance, crack) is determined by the well-known formula [19]:

$$\frac{dM_i}{d\tau} = -\mu \frac{\rho f_i}{\sqrt{RT}} \left(\frac{p_0}{p} \right)^{\frac{1}{\gamma}} \sqrt{\frac{2\gamma}{\gamma-1} \left[1 - \left(\frac{p_0}{p} \right)^{\frac{\gamma-1}{\gamma}} \right]} \quad (14)$$

where μ is the flow coefficient [22]; f_i is the cross-sectional area through which the flow or leak occurs (for an engine cylinder, in addition to the leakage area, it is necessary to specify the valve opening dependence based on the crankshaft rotation angle); p_0 is the ambient pressure or the pressure in the channels adjacent to the cylinder. Thus, when using formulas (9)-(14), the system of equations (6) is a mathematical model and can be solved numerically with the initial conditions: at $\tau=0$ $\phi=0, x=0, T=T_0, p=p_0$. For the solution, you can use the simple or modified Euler method (second-order Runge-Kutta method [23]).

Result and Discussion

The developed model was used to solve three completely different applied problems, including:

A. Modeling a hydrolock when liquid enters the cylinder of an internal combustion engine. The compression process was mod-

eled with different filling coefficient of the combustion chamber with liquid in order to determine the conditions for losing stability (buckling) of the connecting rod of a given cross-section (Figure 3a & 3b)), which allows us to determine the limits of engine performance when operating conditions are violated.

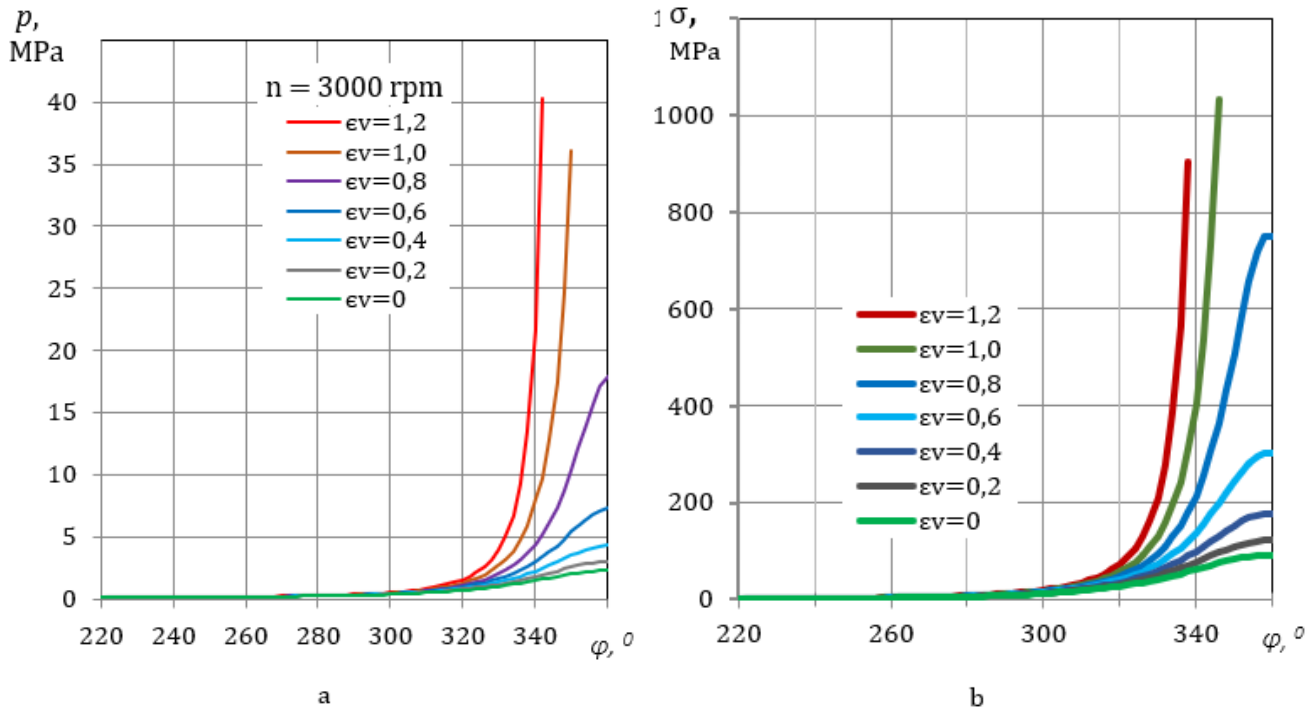


Figure 3: General diagram of the studied “cylinder-piston” system: 1-piston, 2-inlet/outlet valve.

B. Modeling engine workflow with determination of indicator diagram during cold cranking when measuring compression (Figure 4a, 4b & 4c). The main objective of the study was to obtain data on the effect of various damages on the compression value and the shape of the pressure curve in the cylinder, which can be used to diagnose the technical condition of engines in operation.

C. Modeling the dynamics of the trolley (carriage) motion during catapult operation when an Unmanned Aerial Vehicle (UAV) starts. This study was carried out to determine the effect of various design factors on the efficiency of the pneumatic launch system in order to subsequently select its optimal parameters (Figure 5a & 5b).

The process of air compression in a cylinder during hydrolock was simulated for different liquid amount, determined using the filling coefficient of the “dead” volume ϵ_v . Calculations showed that the liquid begins to affect the parameters of the working fluid in the cylinder during the piston’s upward stroke, starting from 45-500 to the top dead center (Figure 3a). During modeling an important result was obtained [11,19]: When the “dead” volume is filled with liquid by more than 80%, the pressure in the cylinder increases so significantly that it causes a sharp increase in loads (stresses in the

parts) in the piston drive mechanism (Figure 3b). This leads to the well-known phenomenon of the connecting rod buckling [18], as well as to other damage to the mating parts [19]. In other words, when setting the ultimate strength of the connecting rod material, which usually does not exceed 800MPa, the model allows us to get the maximum permissible amount of liquid in the cylinder determined by the filling coefficient $\epsilon_v = 0.8$. When this amount is exceeded, the conrod buckling appears and the engine fails. Such data are important for the practical operation of various vehicles when searching for the causes of engine damage [11]. Modeling of cold engine cranking was performed for various types of damage to the cylinder-piston and valve group (Figure 4a). This study revealed a number of patterns in the change in compression depending on the type and size of engine damage, as well as on the crankshaft rotation speed, which is determined by the condition of the starter, battery, engine temperature, etc. Such patterns can be obtained experimentally, but the volume of experimental work, especially for different engines, becomes so significant that it makes such work practically impossible [9]. As a result, modeling for the purpose of determining such patterns has virtually no alternative if we consider not a single engine of a specific model, but a wide range of engines [11].

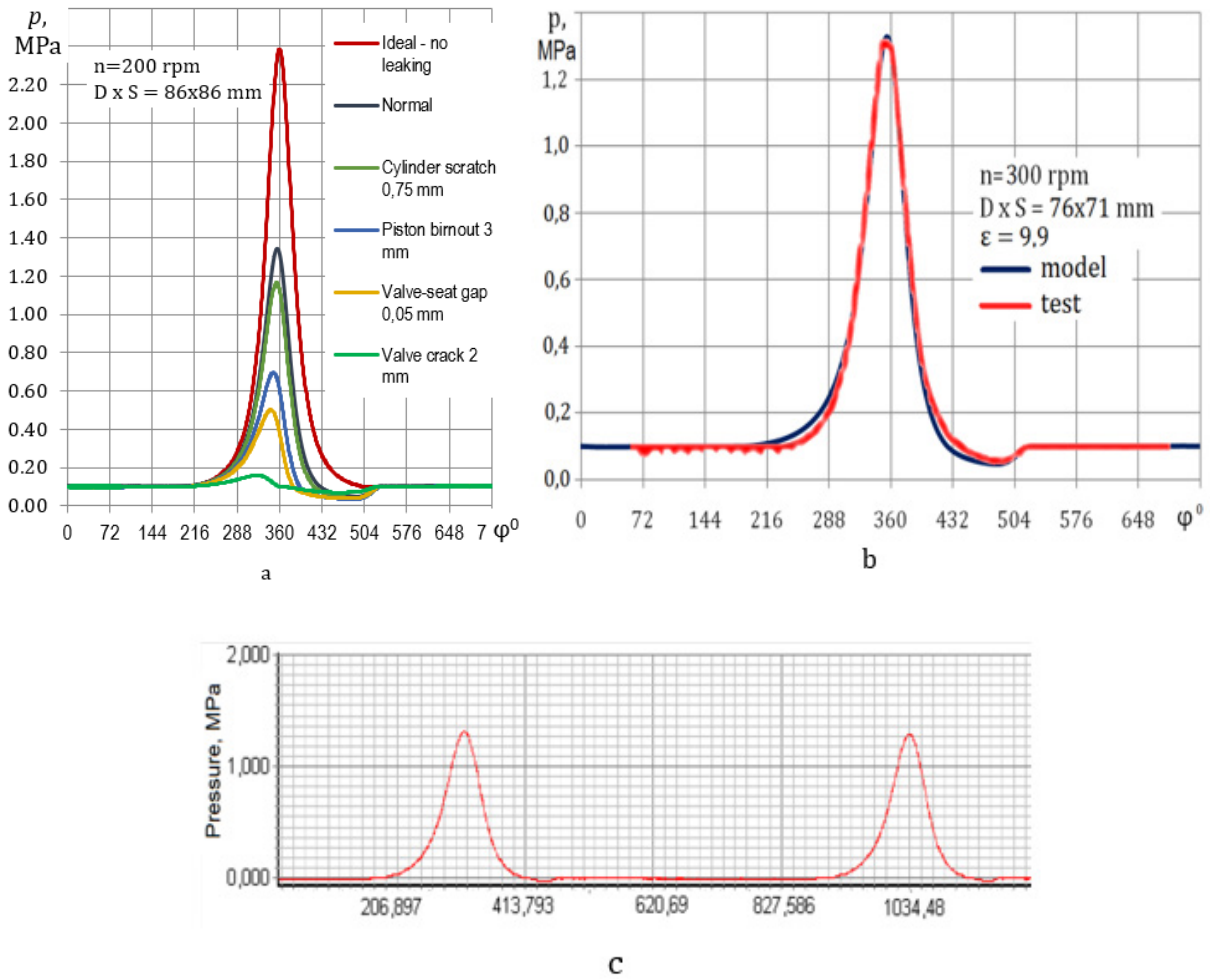


Figure 4: Results of modeling the engine indicator diagram during cold cranking when measuring compression for various types of damage (a) and comparison of the calculated diagram (b), with known data [9] from compression tests (c).

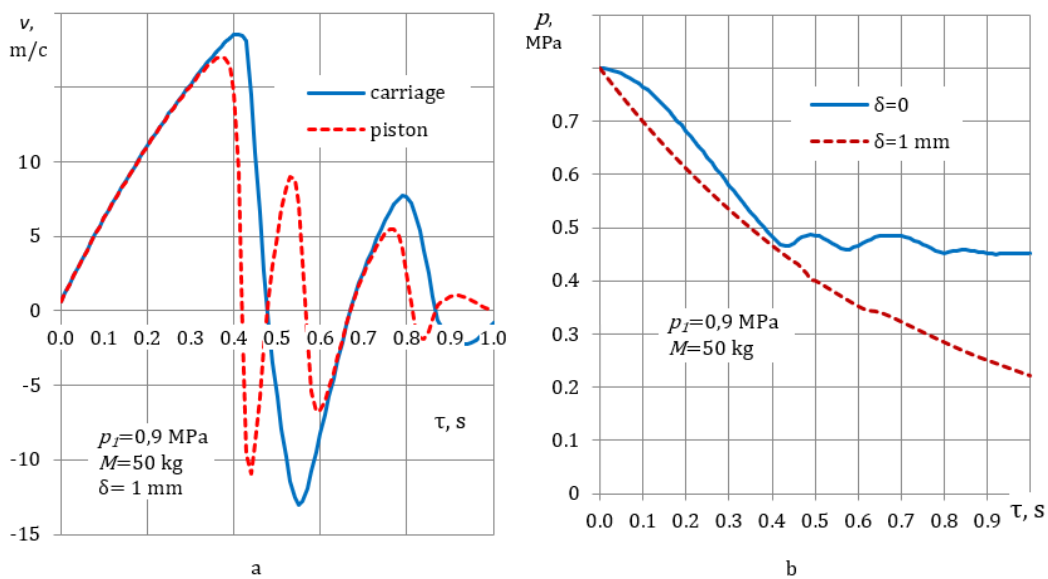


Figure 5: Change of the trolley (carriage) and piston velocity over time (a) and the effect of air leakage from the cylinder on the change in system air pressure (b).

Of particular importance for practice is also data on the shape of the pressure diagram during cold engine cranking (Figure 4b). During the research, it was noted that the size of the so-called "pocket" on the bottom of the diagram showing the vacuum section in the cylinder, directly depends on the air leak during the compression stroke [9]. The influence of the pattern and size of the damage on the dimensions of the "pocket" was revealed from the results of parametric modeling, which showed that the more air leaks from the cylinder, the wider and deeper the "pocket" on the diagram. At the same time, good concordance between the calculated data and previously conducted experimental tests [14,15] for several types of engines with various damage (Figure 4b & 4c), confirms the reliability of the proposed model. The application of the developed thermodynamic model for simulating the acceleration and deceleration processes of the catapult trolley is of particular interest. On the one hand, the results of the modeling demonstrate the model universality and its applicability for describing processes in completely different piston-type devices. Thus, as a result of the modeling, diagrams of velocities (Figure 5a), accelerations and forces acting in this system were obtained, and the influence of various design factors on the efficiency of its operation was determined [20]. On the other hand, the model allows us to identify such patterns of processes that are inaccessible to any other models, especially to those built on a simple analysis of the acting forces. For example, using the model, it is possible to determine the influence of leaks from the working cylinder (through the piston gap in the cylinder) on the trolley acceleration and velocity during the start (Figure 5b). The influence of other design parameters, for example, the pulley diameter and its location relative to the cylinder, the mass of the load on the trolley, the UAV start speed and other parameters, is modeled similarly [21]. From the obtained results, it follows that the developed universal thermodynamic model allows solving a wide range of applied tasks by workflow modeling in various types of piston-type devices and engines, including both their working and non-working modes caused by breakage of the operating conditions. At the same time, as the obtained results show, the model can be used not only in operational problems, in diagnostics of technical conditions and search for failure causes, but also in the design of new piston-type devices.

Conclusion

A universal thermodynamic model for calculating the workflow of "cylinder-piston" type systems which describes step by step the processes in the cylinder, and takes into account the real nature of the intake-exhaust and air leakage processes has been developed. Using the model, three completely different applied problems, including modeling hydrolock when fluid enters the cylinder of an internal combustion engine, modeling the dynamics of the catapult trolley motion during the launch of an unmanned aerial vehicle, and modeling the engine indicator diagram during cold cranking when measuring compression were solved. Control modeling and comparison with known experimental data on the processes of air compression in cylinder with liquid (hydrolock), results of pressure measures in the engine cylinder during cold cranking and the

known data of pneumatic launchers (catapults) showed the model's overall reliability and its applicability for various applied tasks. As a prospect for further research, it is proposed to improve the model and expand it to other types of piston devices, as well as continue the study of the model itself with the aim of its possible application not only in designing devices of the type under consideration, but also for their operational diagnosing and troubleshooting of various types of damage.

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